

# Semester Two Examination, 2023

# **Question/Answer booklet**

# MATHEMATICS APPLICATIONS UNITS 3&4

# Section Two: Calculator-assumed

WA student number: In fi

In figures

In words

SOL	UTI	ONS	

Time allowed for this section Reading time before commencing work: Working time:

ten minutes one hundred minutes

# Materials required/recommended for this section

**To be provided by the supervisor** This Question/Answer booklet Formula sheet (retained from Section One)

## To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators, which can include scientific, graphic and Computer Algebra System (CAS) calculators, are permitted in this ATAR course examination

# Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	52	35
Section Two: Calculator-assumed	12	12	100	99	65
				Total	100

### Instructions to candidates

- 1. The rules for the conduct of Trinity College examinations are detailed in the *Instructions to Candidates* distributed to students prior to the examinations. Sitting this examination implies that you agree to abide by these rules.
- 2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.
- 3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.
- 4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.
- 5. It is recommended that you do not use pencil, except in diagrams.
- 6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.
- 7. The Formula sheet is not to be handed in with your Question/Answer booklet.

#### Section Two: Calculator-assumed

This section has **twelve** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

65% (98 Marks)

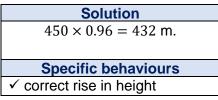
SEMESTER TWO 2023 CALCULATOR-ASSUMED

#### Question 8

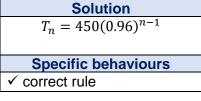
(7 marks)

A weather balloon is launched from the ground and rises a vertical distance of 450 m during its first minute of flight. In each subsequent minute, it rises 96% as far as in the previous minute.

(a) Determine the height that the balloon rises in the second minute of flight. (1 mark)

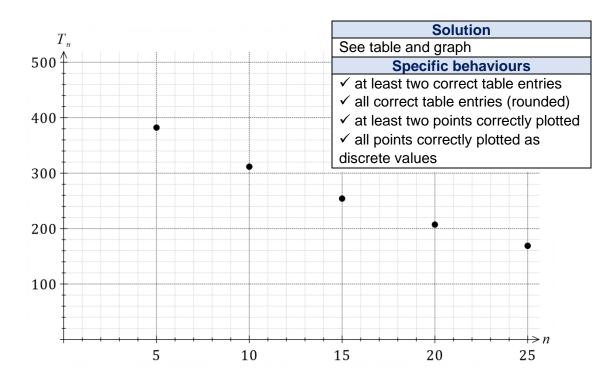


(b) Deduce a rule for  $T_n$ , the rise in height of the balloon in metres during the  $n^{\text{th}}$  minute of its flight. (1 mark)



(c) Complete the following table to show the rise in height (to the nearest metre) during the specified minutes and display the points from the table on the axes provided. (4 marks)

Minute of flight, n	5	10	15	20	25
Rise in height, $T_n$	382	312	254	207	169



(d) During which minute is the rise in height of the balloon closest to 60 m?

(1 mark)

Solution				
During the 50 <sup>th</sup> minute. ( $T_{50} = 60.8$ m)				
Specific behaviours				
✓ correct minute				

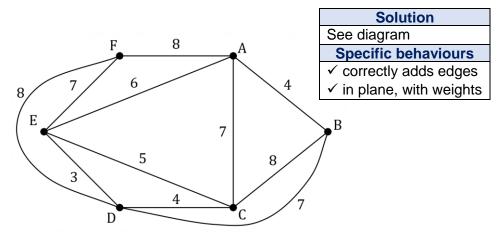
#### See next page

#### (6 marks)

The length of cable required to connect routers A to F is shown in the table below. It is not possible to connect all routers to each other and such pairs are shown with a dash. The network cable used costs \$9 per metre.

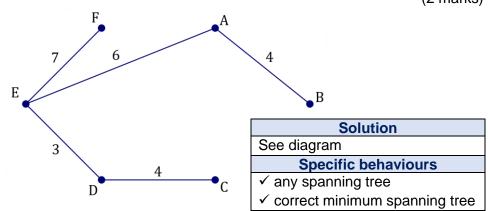
	F	E	D	С	В
А	8	6	Ι	7	4
В	-	-	7	8	
С	-	5	4		
D	8	3			
E	7				

(a) Use the vertices below to complete, in the plane, an undirected weighted graph that represents the information in the table. (2 marks)



(b) Use the vertices below to draw the minimum spanning tree for the graph in part (a).

(2 marks)



(c) If it is no longer possible to connect routers A and B, determine the cost of cable to connect the routers using the new minimum spanning tree. (2 marks)

 Solution

 New spanning tree: drop AB and use DB, so length of MST is 7 + 6 + 3 + 4 + 7 = 27 m.

 Cost will be  $27 \times 9 = $243$ .

 Specific behaviours

 ✓ indicates change in spanning tree

 ✓ correct length of tree / cost

**SEMESTER TWO 2023** CALCULATOR-ASSUMED

#### **Question 10**

#### (7 marks)

The number of rainy days at a weather station were recorded and a summary by quarter for a 3-year period is shown in the table below.

Year		20	20			20	21			20	22	
Quarter	1	2	3	4	1	2	3	4	1	2	3	4
Days	18	10	23	34	15	9	24	38	23	Α	22	33

(a) Determine

> the 3-point moving average for the fourth quarter of 2020. (i) (1 mark)

	0	0	
			Solution
(2	23 +	34 + 1	$5) \div 3 = 72 \div 3 = 24$
		Specif	fic behaviours
✓ corr	ect n	noving	average

(ii) the 4-point centred moving average for the second quarter of 2021. (1 mark)

Solution
$$\left(\frac{34}{2} + 15 + 9 + 24 + \frac{38}{2}\right) \div 4 = 84 \div 4 = 21$$
Specific behaviours $\checkmark$  correct moving average

the value of A in the table, given that the 5-point moving average for the first (iii) quarter of 2022 is 23. (2 marks)

Solution
$$(24 + 38 + 23 + A + 22) \div 5 = 23$$
 $107 + A = 115$  $A = 8$ Specific behaviours $\checkmark$  uses given values to correctly form equation $\checkmark$  correct value of A

(b) Briefly explain the purpose of calculating moving averages for time series data. (1 mark)

Solution					
To smooth / remove seasonality from / expose the trend in the data.					
Specific behaviours					
✓ indicates valid purpose					

Which is the most appropriate type of moving average for use with the data in the table (c) above? Justify your choice. (2 marks)

Solution					
Most appropriate is 4-point centred, due to the annual					
data being split into 4 quarters / having a 4-point cycle.					
Specific behaviours					
✓ correctly states appropriate type					
$\checkmark$ supplies a valid justification					

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#### Question 11

### (7 marks)

A finance company offers three loan products to borrowers, with interest compounded at different intervals:

Product	Х	Y	Z
Annual interest rate	8.68%	8.70%	8.77%
Compounding period	Daily	Monthly	Quarterly

(a) Which product would minimise the interest payable on a loan taken out for two years? Justify your answer. (3 marks)

Effective rates for each product using financial calculator:

Product X: N = 365, APR = 8.68, EFF = 9.067%

Product Y: N = 12, APR = 8.70, EFF = 9.055%

Product Z: *N* = 4, *APR* = 8.77, *EFF* = 9.063%

Product Y will minimise interest payable as it has the lowest effective interest rate.

#### Specific behaviours

- $\checkmark$  correctly calculates one effective interest rate
- ✓ correctly calculates all effective interest rates
- ✓ draws correct conclusion
- (b) Determine the interest that a borrower will have to pay on a loan of \$2500 for nine months using product Z. (2 marks)

Solution					
$(1.8.77)^{3}$	Using financial calculator:				
$A = 2500 \left( 1 + \frac{8.77}{4 \times 100} \right)^3$	N = 3, I% = 8.77, PV = 2500				
A = 2668.07	PMT = 0, P/Y = 4, C/Y = 4				
	Solve for $FV = -2668.07$				
Hence \$168.07	nterest charged.				
Specific behaviours					
✓ shows compound interest formula or values used in financial calculator					

- ✓ correct amount of interest
- (c) A borrower took out a short-term loan of \$2200 dollars using product X. If they paid \$19.44 interest on the loan, determine the duration of the loan in days. (2 marks)

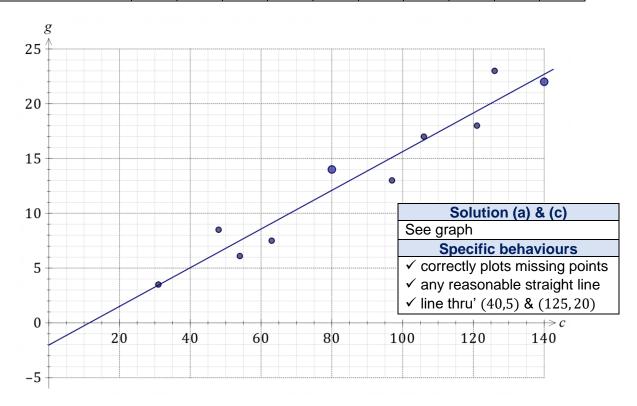
SolutionUsing financial calculator:I% = 8.68, PV = 2200, PMT = 0, FV = -2219.44P/Y = 365, C/Y = 365Solve for N = 37 days.Specific behaviours $\checkmark$  shows values used in financial calculator $\checkmark$  correct number of days

#### See next page

#### (10 marks)

The capacity of a solar farm is the maximum amount of power the farm is the designed to produce. The table below shows the power capacity in megawatts of ten Australian solar farms (labelled A to J) and the total power each generated in gigawatt-hours over the course of October last year.

Solar farm	А	В	С	D	Е	F	G	Н	I	J
Capacity, c MW	31	80	106	48	121	63	140	126	54	97
Generation, g GWh	3.5	14	17	8.5	18	7.5	22	23	6.1	13



- (a) Use data from the table to plot the points B and G on the graph. (1 mark)
- (b) Determine the equation of the least-squares line in the form  $\hat{g} = ac + b$ , where *a* and *b* are constants, and state the value of the correlation coefficient. (2 marks)

Solution
$$\hat{g} = 0.177c - 2.03, \quad r = 0.967$$
Specific behaviours $\checkmark$  correct variables and coefficients for line $\checkmark$  correct value of  $r$ 

(c) Draw the least-squares line on the graph.

(2 marks)

(d) Interpret the gradient of the least-squares line in the context of the question. (1 mark)

Solution
For every 1 MW increase in capacity of a solar farm,
the power it generated increased by 0.177 GWh.
Specific behaviours
✓ correct interpretation

What percentage of the variation in power generation can be explained by the variation in (e) power capacity? (1 mark)

Solution
$r^2 = 0.935$
93.5% of the variation.
Specific behaviours
✓ correct percentage

Australian solar farm K, with a capacity of 74 MW, was also operating last October.

(f) Predict the total power generated by solar farm K that month. (1 mark)

Solution	
g = 0.177(74) - 2.03	
= 11.0 GWh	
Specific behaviours	
✓ correct prediction	

(g) Give two reasons why the prediction in part (f) is reliable. (2 marks)

Solution	
The correlation coefficient indicates a very relationship between the variables.	strong
The prediction involves interpolation (does involve extrapolation).	not
Specific behaviours	
✓ strength of association	
<ul> <li>✓ interpolation (or no extrapolation)</li> </ul>	

A small business has organised a loan of \$6250 to purchase a new machine. To pay off the loan, the business will make repayments of \$233.08 at the end of each month. The spreadsheet below shows how the balance of the loan will reduce in the first few months.

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Month (n)	Opening balance $(T_n)$	Interest	Repayment	Closing balance	
1	6250.00	46.25	233.08	6063.17	
2	6063.17	44.87	233.08	5874.96	
3	5874.96	43.47	233.08	5685.35	

#### (a) Show that the loan interest rate is 8.88% per annum.

Solution Monthly rate:  $46.25 \div 6250 = 0.0074$ 

Annual rate is  $0.0074 \times 12 = 8.88\%$ .

 Specific behaviours

 ✓ indicates correct calculation for monthly rate

 ✓ indicates correct calculation for annual rate

 See spreadsheet

 Specific behaviours

 ✓ opening balance & repayment

 ✓ interest

 ✓ closing balance

The recursive rule for the opening balance is  $T_{n+1} = aT_n - b$ ,  $T_1 = c$ , where  $T_n$  is the opening balance of the loan in month *n*.

(c) Determine the values of the constants *a*, *b* and *c*.

Solutiona = 1 + 0.0074 = 1.0074b = 233.08, c = 6250Specific behaviours $\checkmark$  correct value for a $\checkmark$  correct values for b and c

(d) Determine how many months it will take the business to fully repay the loan. (1 mark)

Solution
Since $T_{31} = 0$ then it took 30 months to fully repay loan.
Specific behaviours
✓ correct number of months

(2 marks)

(e) Calculate the total interest paid by the business once the loan has been fully repaid.

(2 marks)

Solution
Total repayments: $30 \times $233.08 = $6992.40$
Hence total interest is \$6992.40 - \$6250 = \$742.40.
Specific behaviours
✓ total repaid
✓ total interest

(f) If the interest rate was doubled but the loan repayments remained the same, would the time to repay the loan also double? Justify your answer. (2 marks)

Solution
No, time would not double.
Time would increase by 5 months, to 35. OR The interest is a relatively small portion of each repayment, so the time would increase but not double.
Specific behaviours
✓ states time would not double
✓ any reasonable justification

**SEMESTER TWO 2023** CALCULATOR-ASSUMED

#### **Question 14**

(11 marks)

National chess organisations rate all players who are registered to compete in tournaments. When a higher rated player competes against a lower rated player, the higher rated player is expected to win. The table below shows the average rating of registered South African players of different ages.

Age, x years	10	11	12	13	14	15	16	17	18
Average rating, y	1216	1196	1218	1292	1253	1282	1319	1306	1354

The data has a correlation coefficient of 0.92 and the equation of the least-squares line is y = 17.9x + 1020.

Explain the significance of the correlation coefficient in the context of the question. (a)

(2 marks)

Solution
The coefficient of 0.92 is positive which means that there is a positive association between average rating and age. The coefficient is also close to 1 which means that the association is strong.
Specific behaviours
a explanation uses both variables

explanation uses both variables

✓ indicates association strength is strong and/or direction is positive

(b) Predict the average rating of 19-year old registered South African players and briefly discuss the validity of your prediction. (3 marks)

Solution y = 17.9(19) + 1020 = 1360.

The predicted average rating of 1360 for 19-year old players required slight extrapolation beyond the upper bound of the given data, and due to the strong correlation, prediction is valid.

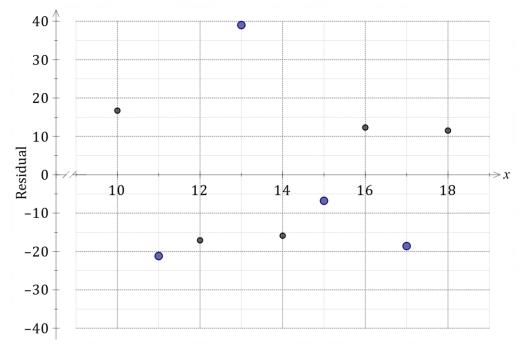
**Specific behaviours** 

- ✓ correct prediction
- ✓ states extrapolation
- ✓ states valid due to strong correlation
- It is unlikely that simply getting older causes players ratings to increase. Suggest another (c) variable that is more likely to cause the rating increase. (1 mark)

Solution
The number of games played / tournaments entered / experience /
coaching / practice / etc.
Specific behaviours
✓ any reasonable variable

(d) Complete the residual plot for the data on the axes below.

(3 marks)



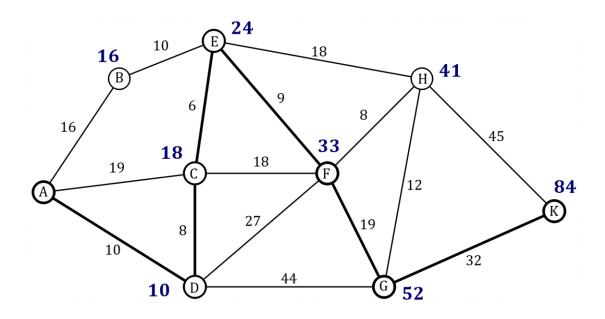
Solution						
	x	у	$\hat{y}$	$y - \hat{y}$		
	11	1196	1217	-21		
	13	1292	1253	39		
	15	1282	1289	-7		
	17	1306	1324	-18		
Specific behaviours						
✓ indicates at least one correct residual						
✓ correctly plots at least two points						
✓ correctly plots all points						

(e) Use the residual plot to comment on the appropriateness of fitting a linear model to the data. (2 marks)

Solution
Fitting a linear model is appropriate as no
pattern is evident in the residual plot.
Specific behaviours
🗸 states linear model is appropriate

- ✓ states linear model is appropriate
- ✓ gives valid reason

The graph represents a network of tracks in a park between nine locations labelled A, B, C, D, E, F, G, H and K. The number on each edge is the length, in hundreds of metres, of the corresponding track. The total length of all tracks in the network is 30 100 m.



(a) Determine the shortest distance along these tracks a person at location D must travel to reach location B and state the corresponding route. (2 marks)

Solution
Following the route D-C-E-B, the edge weights are $8 + 6 + 10 = 24$
and so the shortest distance is 2400 m.

NB Penalise missing hundreds once in question.

Specific behaviours

- ✓ correct route
- ✓ correct length of route
- (b) Determine the shortest path from location A to location K and state the length of this path.

(4 marks)

See network. Numbers beside each vertex are the shortest distance from A to that vertex.

Shortest path: A-D-C-E-F-G-K.

Edge weights: 10 + 8 + 6 + 9 + 19 + 32 = 84.

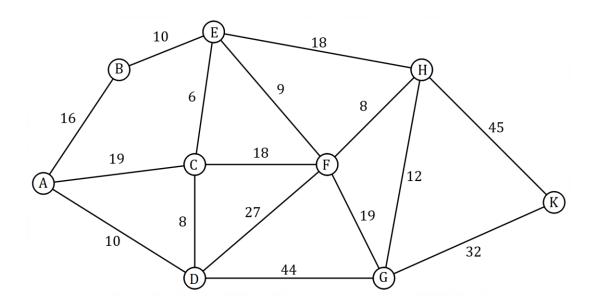
Length of shortest path is 8400 m.

#### Specific behaviours

- ✓ states a correct path from A to K
- ✓ states length of a correct path from A to K
- ✓ correctly determines shortest path
- ✓ correct length of shortest path

#### See next page

A copy of the network from the previous page is shown below.



(c) Every track in the park is to be inspected by a ranger who must start and end the inspection at location A and travel along each track at least once. Determine the least distance the ranger must travel to perform this task. Justify your answer. (3 marks)

#### Solution

The graph has two odd vertices, one at A and the other at F, and hence it contains an Eulerian trail. The trail starts at A, ends at F and has length 30 100 m (given at start of question).

The ranger must then return from F to A via the shortest path which is E-C-D, 9 + 6 + 8 + 10 = 33.

Hence the least distance is  $30\ 100 + 3300 = 33\ 400$  m.

Specific behaviours

- ✓ indicates use of Eulerian trail
- ✓ indicates shortest path from F to A
- $\checkmark$  correct least distance

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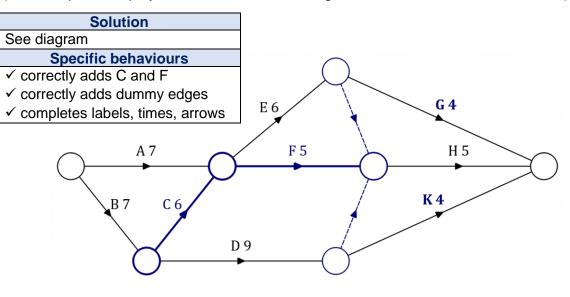
#### **Question 16**

#### (7 marks)

The table below shows the nine activities that must be completed in order to assemble a piece of flat-packed furniture, together with the time required and immediate predecessors for each activity involved in the project.

Activity	А	В	С	D	E	F	G	Н	K
Time (minutes)	7	7	6	9	6	5	4	5	4
Immediate predecessors	_	-	В	В	A, C	A, C	Е	E, F, D	D

(a) Complete the project network below, showing all activities and their times. (3 marks)



(b) Determine the critical path and the minimum completion time for the project. (2 marks)

Solution
Critical path is $B - C - E - H$ and minimum
completion time is $7 + 6 + 6 + 5 = 24$ minutes.
Creatifia hakaviaura
Specific behaviours
✓ correct critical path

✓ correct MCT

(c) A redesign of the flat-packed furniture will reduce the time required for Activity C to just two minutes. What effect will this have on the critical path and the minimum completion time? (2 marks)

Solution			
Critical path changes to B – D – H and minimum			
completion time reduces to $7 + 9 + 5 = 21$ minutes.			
Specific behaviours			
✓ correct new critical path			
✓ correct new MCT (or reduction in time)			

### (7 marks)

The number of residential solar panel installations carried out by a company during each of the first nine months of the year 2020 is shown in the table below.

Quarter	Month	+	Installations	Quarterly	Installations as
Quarter	MOHUI	t	n	average	percentage of average
1	January	1	132		85.2
1	February	2	171	155	111.3
1	March	3	162		104.5
2	April	4	148		87.1
2	May	5	188	Α	110.6
2	June	6	174		102.4
3	July	7	156		86.7
3	August	8	198	180	В
3	September	9	186		103.3

The long-term trend y in the number of installations can be modelled using y = 3.77t + 149.5, and the seasonal index for the last month of every quarter is 1.034.

(a) Calculate the value of *A* and the value of *B* in the table.

(2 marks)

Solution						
$A = (148 + 188 + 174) \div 3 = 170,$	$B = 198 \div 180 \times 100 = 110$					
Specific beha	aviours					
$\checkmark$ correct value of A						
$\checkmark$ correct value of <i>B</i>						

(b) Clearly show why the seasonal index for the first month of every quarter is 0.863. (1 mark)

Solution  
$$SI = (0.852 + 0.871 + 0.867) \div 3 = 2.59 \div 3 = 0.863$$

Specific behaviours ✓ clearly uses correct figures in calculation

(c) Determine the deseasonalised number of installations carried out in March 2020.

(2 marks)

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Solution
Seasonal index for middle month is \frac{1.04+1.024+1.033}{2} = 1.032.
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Deseasonalised number is  $162 \div 1.032 = 157$  installs.

#### Specific behaviours

✓ calculates seasonal index

- ✓ correct deseasonalised value
- (d) Assuming that existing trends and seasonality continue, predict the number of installations carried out by the company during April 2021. (2 marks)

	Solution
t = 16,	$y = 3.77(16) + 149.5 = 209.82,$ $n = 288.05 \times 0.863 = 181.07$
	Predict number of installs to be 181.
	Specific behaviours
✓ calculates t	trend
✓ correct pree	diction See next page

SEMESTER TWO 2023 CALCULATOR-ASSUMED

#### Question 18

(7 marks)

(1 mark)

Lucky has a savings account with a balance of \$402 and plans to deposit \$11 into it on the first day of every month. Interest on the balance of the account, at a rate of 0.6% per month, is added on the last day of every month.

To model the balance of this account, Lucky uses the recurrence relation  $T_{n+1} = 1.006T_n + 11$ ,  $T_0 = 402$ , where  $T_n$  is the account balance in dollars just after he has made his  $n^{\text{th}}$  deposit.

After the ninth deposit, the balance of the account is \$525.65.

(a) Determine the account balance after the  $10^{\text{th}}$  deposit.

Solution $T_{10} = 1.006 \times 525.65 + 11 = $539.80$ Specific behaviours $\checkmark$  correct balance, to nearest cent

(b) Does the sequence generated by the recurrence relation have a long-term increasing, decreasing or steady-state solution? Justify your answer. (2 marks)

Solution			
The sequence continues to increase in the long-term, as the difference			
between consecutive terms is increasing at an increasing rate.			
Specific behaviours			
✓ states increasing			
8			
✓ justifies using increasing differences			

(c) Consider the case where the account paid no interest. Determine the regular monthly deposit Lucky should make into his savings account so that he still achieves the same balance as in part (a) after making 10 deposits.

Solution

Hence require monthly deposit of  $137.80 \div 10 = $13.78$ .

#### **Specific behaviours**

✓ indicates increase in balance required
 ✓ correct monthly deposit

Increase in balance is 539.80 - 402 = 137.80.

(d) Consider the case where Lucky made no monthly deposits. Determine the monthly interest rate required for his savings account so that he still achieves the same balance as in part (a) after 10 compounding periods. (2 marks)

SolutionUsing financial app, solve for interest rate (I%) to grow PV = -402 to FV = 539.80in N = 10 time periods with PMT = 0, P/Y = C/Y = 1.Hence monthly rate is 2.99%.Specific behaviours $\checkmark$  indicates appropriate method $\checkmark$  correct rate

#### Question 19

(9 marks)

Penny has decided to take some time off work and has \$72 500 in an investment account that attracts interest at a rate of 4.65% compounded monthly. She will withdraw living expenses of \$3900 from the account at the end of each month, just after interest is added.

(a) Determine the balance of her account after one year, when Penny has made 12 withdrawals.

(3 marks)

Solution	
Using the financial app:	
$N = 12, I\% = 4.65, PV = -72500, PMT = 3900, PY = 12, CY = 12 \rightarrow FV = 28133.61$	Ĺ
The balance will be \$28 133.61.	

#### Specific behaviours

- ✓ states at least 4 correct entries
- $\checkmark$  states all correct entries
- ✓ states correct balance

(b) Determine the total interest Penny's account will earn during the first year.

(2 marks)

SolutionBalance and withdrawals:  $28 \ 133.61 + 12 \times 3900 = 74 \ 933.61$ Total interest is  $74 \ 933.61 - 72 \ 500 = $2433.61$ .

Specific behaviours

- $\checkmark$  indicates balance and withdrawal total
- $\checkmark$  states correct interest
- (c) For how many months will Penny be able to withdraw \$3900?

(1 mark)

**Solution** Using financial app with settings as in part (a) but solving for N when FV = 0, she can make withdrawals for 19 months.

#### Specific behaviours

✓ correct number of months

(d) Penny contemplated receiving a quarterly perpetuity from her investment account to fund her time off work. Determine the quarterly amount that she would receive. (3 marks)

Solution
Using the financial app:
$N = 1, I\% = 4.65, PV = -72500, FV = 72500, PY = 4, CY = 12 \rightarrow PMT = 846.08$
The amount will be \$846.08.
NB Setting for N can be any number of instalments.
Specific behaviours
✓ states at least 4 correct entries
✓ states all correct entries
✓ states correct amount